

Market dynamics after large financial crash

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(Dated: December 5, 2008)

The model describing market dynamics after a large financial crash is considered in terms of the stochastic differential equation of Ito. Physically, the model presents an overdamped Brownian particle moving in the nonstationary one-dimensional potential U under the influence of the variable noise intensity, depending on the particle position x . Based on the empirical data the approximate estimation of the Kramers-Moyal coefficients $D_{1,2}$ allow to predicate quite definitely the behavior of the potential introduced by $D_1 = -\partial U/\partial x$ and the volatility $\sim \sqrt{D_2}$. It has been shown that the presented model describes well enough the best known empirical facts relative to the large financial crash of October 1987.

PACS numbers: 89.65.Gh, 02.50.Ey, 05.40.-a

I. INTRODUCTION

The dynamics of the financial markets has been attracting attention of the physics community for two decades [1, 2, 3, 4]. During this time a large volume of the empirical researches has been done. Of specific interest is the investigation of the behavior of the market in the time periods of the large financial crises when the statistical properties of the market are drastically distinguished from its properties in quiet days [3, 5, 6, 7]. The empirical analysis has found the occurrence of a number of peculiarities in market dynamics appearing in these periods. Originally they have been revealed when investigating the large financial crash on 19 October 1987 (Black Monday) at New York Stock Exchange. However these peculiarities are not specific for October 1987 and have been later revealed in the financial crashes of 1997 and 1998 years for different countries and Exchanges [7].

Firstly, it has been established that the large financial crashes are outliers which are not possible within the scope of the typical price distributions. For their realization the complementary factors absent in quiet days are needed [5, 7].

Secondary, the emergence of the certain periodic patterns in the dynamics of the different financial assets are detected both before the crash, and immediately after it. In particular, a log-periodic oscillations in time evolution of the price of an asset appear before the extreme event [7, 8, 9, 10, 11, 12]. The aftershock period is characterized by an exponentially decaying sinusoidal behavior of a price [7, 8]. The characteristic behavior of volatility is also revealed after the crash. In the moment of the crash of October 1987 the implied volatility of the S&P500 index makes a shock and then sharply decays as power law decorated with log-periodic oscillations [7, 8].

In the day of the maximum drop the central part of the empirical return distribution moves toward negative returns and then begins to oscillate between positive and negative returns [6].

At last, the relaxation dynamics of an aftercrash period is characterized by what is termed as the Omori law which shows that the rate of of the return shocks larger than some threshold decays as the power law with exponent close to 1 for different thresholds [13, 14].

There are a large number of works devoted to the modeling of the market dynamics immediately before crash [7, 8, 9, 10, 11, 12, 15, 16] (see also references in the article of Sornette [7]). These works based on the empirical observations exploiting the analogy with critical phenomena have been directed to the investigation of a possibility of predictions of the large financial crash on the base of the modeling of the dynamics of different financial indexes (S&P500, DJ, etc).

The "microscopic" approach has been suggested in the work [17] where the model of the market dynamics taking into account a possibility of occurrence of the crash is developed. In physical terms, the model is reduced to a motion of a Brownian particle in the stationary cubic potential and crashes are considered as rare activated events.

In Refs. [18, 19] the generalization of the above model has been suggested for the case of the stochastic volatility which was considered within the scope of the modified Heston model. Physically, the model represents an overdamped Brownian particle moving in the stationary cubic potential under the influence of the fluctuating noise intensity.

Since the statistic properties of the volatility in the Heston model are uniform in a time [20], both the above models describe the uniform stochastic process. In financial terms, this means that statistic properties of the market are the same regardless of where market is either in normal regime or close to a crash. It seems this is poorly consistent with the assertions that crashes are outliers and the statistic properties of the market are

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drastically change in extreme days [5, 7]. While within the scope of Brownian motion the assertion that crashes are outliers and strong nonstationarity of the market revealed in extreme days [6] lead to the conclusion that an "external" time-dependent potential perturbing the stationary potential profile, in which a fictitious Brownian particle moves in the case of typical days, is triggered in the vicinity of the crash. It is likely the appearance of such potential could be traced developing the microscopic approach in the spirit of the work [17]. It is worth saying that the authors of this work have also noted that in the vicinity of a crash additional market mechanisms, which have not been considered in [17], must be taken into account to remove divergences emerging in their model of the aftershock dynamics.

Another way to detect the influence of the "external" nonstationary perturbation is to analyze the empirical data concerning the extreme events. It is such approach that will be considered in the given work for the modeling of the market dynamics during the large financial crash of October 1987.

The paper is organized as follows. In Sec.II in terms of the stochastic differential equation (SDE) Ito we consider the model describing the market dynamics during a crash. Sec.III is devoted to the analysis of the empirical data and the estimation of the Kramers-Moyal coefficients. In Sec.IV the discussion of results and conclusion are given.

II. THE MODEL

We consider the model in which immediately before a crash and in the short aftercrash period the dynamics of a financial assets is described by the SDE Ito:

$$dx = D_1(x, t)dt + \sqrt{D_2(x, t)} dW(t), \quad (1)$$

$$D_1(x, t) = -\frac{\partial U(x, t)}{\partial x}, \quad (2)$$

where $x(t) = \ln S(t+1)/S(t)$ is the daily log-return, $S(t)$ is the price of the asset at time t , $W(t)$ is a standard Wiener process, D_1 and D_2 are the drift coefficient and the diffusion coefficient, respectively. Physically, Eq. (1) describes a motion of an overdamped Brownian particle in a potential $U(x, t)$ [21], introduced by Eq. (2), with a variable noise intensity depending on a particle position. Like the specific models considered in [17, 18, 19] Eq. (1) presents a Markovian process.

As it has been noted above in the moment of the crash of October 1987 the implied volatility of the S&P500 index makes a shock with subsequent relaxation to its beforecrash level during several weeks. Simultaneously after the index decline, its "stabilization" is approximately reached within the same time [7, 8]. All this allows to consider that a peculiar synchronization of their movements occurs during the crash. In turn, this circumstance makes it possible to assume that in this period the asset

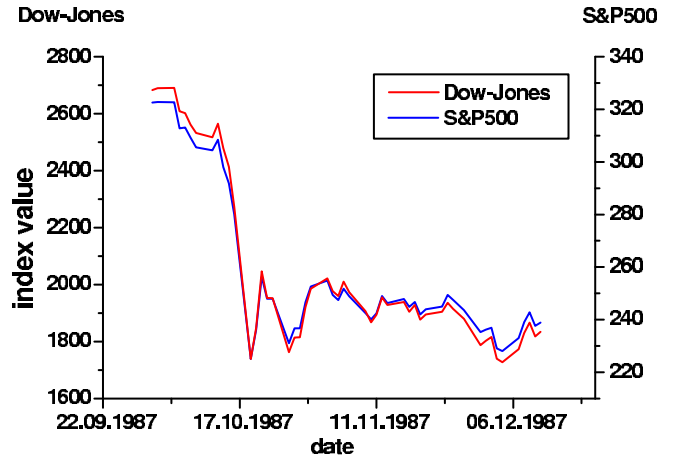


FIG. 1: The time evolution DJ and NASDAQ indexes.

volatility can be considered as a function of the return x and employ the one-dimensional model (1) with the variable diffusion coefficient $D_2(x, t)$ being a measure of the volatility.

As it is known [21] the coefficients D_1 and D_2 from SDE (1), defining the stochastic process $x(t)$, coincide with the Kramers-Moyal coefficients and are given by the equations

$$D_k(x, t) = \lim_{\tau \rightarrow 0} \frac{1}{\tau} M_k(x, t, \tau) \quad (k = 1, 2) \quad (3)$$

$$M_k(x, t, \tau) = \int dx' (x' - x)^k P(x', t + \tau | x, t), \quad (4)$$

where $P(x', t + \tau | x, t)$ is the conditional probability distribution function of $x(t)$.

Originally, within a financial context, the method of the estimation of the Kramers-Moyal coefficients directly from the empirical data has been given in Refs. [22, 23]. Further this method has been applied to different financial assets and exchange rates [24, 25, 26, 27, 28, 29, 30]. In all cases the underlying stochastic process is supposed to be stationary (uniform) and the reconstruction the drift coefficient D_1 from the empirical data gave the linear dependence on corresponding return. According to Eq. (2) this leads to the parabolic potential for a fictitious particle that is typical for the quiet market periods. Here we shall consider the estimation of the coefficients D_k for the case of the extreme events of October 1987 when the market was in a substantially nonstationary phase [6].

III. THE ESTIMATION OF KRAMERS-MOYAL COEFFICIENTS

"From the opening on October 14, 1987 through the market close on October 19, major indexes of market valuation in the United States declined by 30 percent or more. Furthermore, all major world markets declined

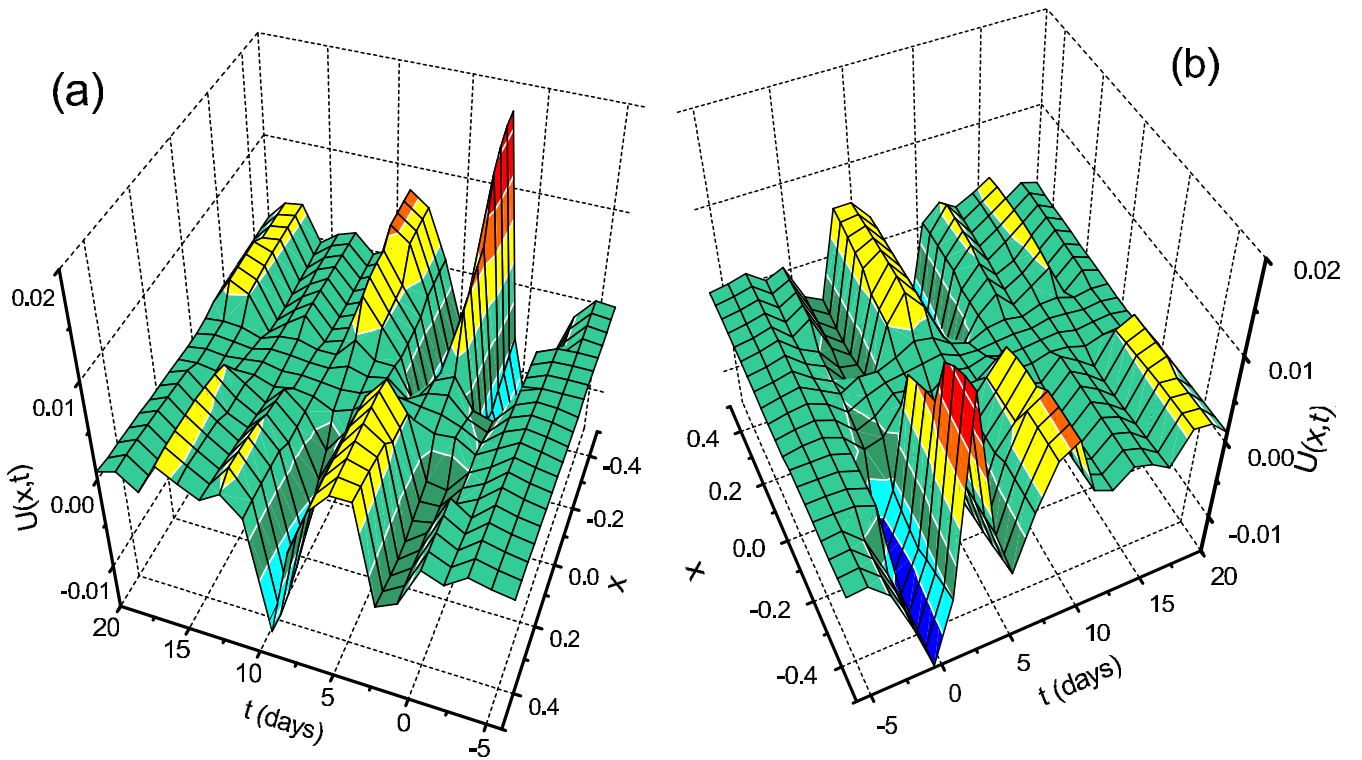


FIG. 2: The reconstruction of the potential U . In the figures (a) and (b) the potential surface is viewed from two mutually opposite sides

substantially in the month, which is itself an exceptional fact that contrasts with the usual modest correlations of returns across countries and the fact that stock markets around the world are amazingly diverse in their organization” [7, 8]. In Fig. 1 the time evolution of two financial indexes DJ and NASDAQ characterizing different sectors of USA economics are given. Practically identical dynamics of these indexes during the crash is seen from Fig. 1. Such behavior is not accidental and is well consistent with the assertion that large financial crashes are caused by appearance of substantial cooperation in the behavior of different markets over the world and local self-reinforcing imitation between traders [7]. These two circumstances led to the substantial synchronization in the dynamics of different financial assets simultaneously traded both in the common market and in different ones, that Fig. 1 shows. These circumstances allow to suggest also that log-returns of different stocks simultaneously traded in markets during the crash have the identical statistical properties and to consider the probability distributions of the whole ensemble of such stocks. Actually this approach has been used in Ref. [6].

In accordance with the above said we consider the ensemble which consists of $n = 467$ stocks traded in Stock Exchanges NYSE, NASDAQ and AMEX in the period from 13 October to 13 November 1987 [31] each charac-

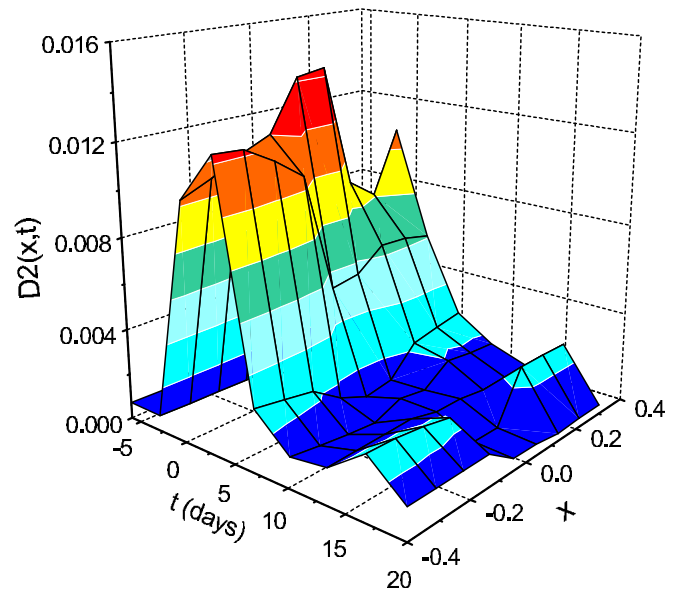


FIG. 3: The reconstruction of the diffusion coefficient D_2 .

terized by the daily log-returns

$$x_i(t) = \ln \frac{S_i(t+1)}{S_i(t)} \quad i = 1, 2 \dots n,$$

where $S_i(t)$ is the close price of i -th asset on day t and the time is measured from the moment of the crash of 19

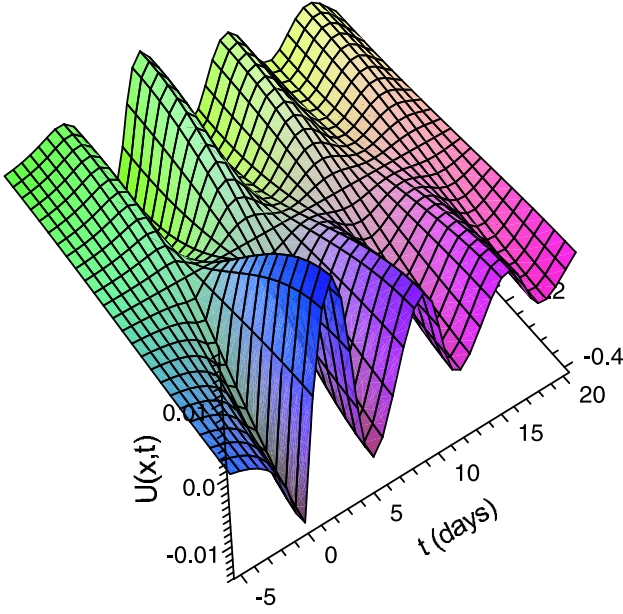


FIG. 4: The potential surface U given by Eq. (7)

October.

Apart from the assumption that during the crash the statistic properties all $x_i(t)$ are identical we shall introduce a fictitious index $S^*(t)$ and consider the empirical data $x_i(t)$ ($i = 1, 2, \dots, n$) as the set of "experimental" values of the log-returns of this index detected at moment t . Further the estimation of the Kramers-Moyal coefficients will be performed just for the index $S^*(t)$ with log-return $x(t)$.

According to Eq. (3) the calculation of the coefficients D_k includes the operation of taking the limit $\tau \rightarrow 0$. For the case of the high-frequency sampling a sufficiently exact estimation can be obtained merely as ratio

$$D_k(x, t) \simeq \frac{M_k(x, t; \tau)}{\tau} \quad (\tau \rightarrow 0) \quad (5)$$

On the long time scale with the minimal step $\tau = 1$ day, Eq. (5) can be broken down. Nonetheless, the rough but, as it will be seen later, giving definite information estimate of the coefficients D_k can be also obtained in this case at $\tau = 1$. Actually the approximation is used, meaning an extension of the linear dependence on the long time scale:

$$M_k(x, t; \tau) = \tau D_k(x, t) \quad (0 \leq \tau \lesssim 1) \quad (6)$$

Thus the estimate of the Kramers-Moyal coefficients has been taken from equality $D_k(x, t) = M_k(x, t; \tau = 1)$. For the calculation of the conditional moments M_k the empirical conditional densities $P(x_2 t_2 | x_1 t_1) = P(x_2 t_2, x_1 t_1) / P(x_1 t_1)$ ($t_2 > t_1$) have been first found where $P(x_2 t_2, x_1 t_1)$ and $P(x t)$ are the joint two-point probability density and the one-point probability density, respectively. Then the numerical integration has been performed in Eq. (4). Once the coefficient D_1 has been found the potential U is determined from Eq. (2). The results of the calculations of U and D_2 are shown in Figs. 2 and 3.

The periodic structure of the potential surface is clearly distinguishable in Fig. 2. In the regions $x > 0$ and $x < 0$ alternation of wells and hills is observed, the sizes of which rapidly decrease with time. The periodic structures of these two regions are displaced in relation to each other so that at the displacement along x axes a well in the range $x > 0$ transfers to a hill in the range $x < 0$ and conversely. At $t = 0$ there is the well of the largest depth in region $x < 0$ corresponding to the moment of the crash. The behavior of the potential surface with indicated features reconstructed from the empirical data is well enough reproduced by the smooth surface presented in Fig. (4) and given by the equation:

$$U(x, t) = \phi(x, t) \begin{cases} A \sin \omega x e^{-\alpha x + \beta_1 t}, & t < 0 \\ B \sin \omega x \sin(\omega_1 t + b) e^{-\alpha x - \beta_2 t}, & t > 0, \end{cases} \quad (7)$$

where

$$\phi(x, t) = 1 - a x^{1/3} e^{-\gamma |t|} \quad (8)$$

The potential U presented by Eq. (7) exponentially decays with time and included trigonometrical functions reproduce the observed periodic structure. Function $\phi(x, t)$ is largely used for the smooth joint of two surfaces from Eq. (7) at $t = 0$ and rapidly goes to one with the increasing $|t|$. The parameters Eqs. (7) and (8) are determined by comparison with the reconstructed surface on the base

of the least-squares method and presented in Table I.

The reconstruction of the diffusion coefficient $D_2(x, t)$ is shown in Fig. (3). Qualitatively the behavior of the diffusion coefficient is quite definite, in spite of the occurring irregularity. At the moment of the crash D_2 makes a shock with subsequent relaxation to its beforecrisis level within a month. In Fig. (5) such behavior is reproduced

TABLE I: Parameters of Eq.7

A	B	ω	ω_1	α	β_1	β_2	a	b	γ
$8.6 \cdot 10^{-3}$	$1.4 \cdot 10^{-2}$	3.9	1.0	0.6	0.5	0.1	0.96	2.5	0.9

TABLE II: Parameters of Eq. (9)

A	B	p
$7.6 \cdot 10^{-3}$	$9.3 \cdot 10^{-4}$	0.57

by the smooth surface with the equation

$$D_2(x, t) = \begin{cases} A t^{-p}, & t > 0, \\ B, & t < 0, \end{cases} \quad (9)$$

As before the parameters of Eq. (9) have been defined by the least-squares method and given in Table II.

IV. DISCUSSION AND CONCLUSION

In previous section we have estimated the Kramers-Moyal coefficients D_1 and D_2 from the empirical data. In spite of the sufficiently rough estimate, the reconstructed surfaces for the potential $U(x, t)$ and the diffusion coefficient D_2 , presented in Figs. 2 and 3, can be quite definitely approximated by the smooth surfaces with the needed properties.

Firstly, it has been noted in Introduction that in the aftercrash period the center of the empirical return distribution oscillates between positive and negative returns [6]. Evidently it is the periodic structure of the potential surface presented in Fig. 4 that provides such behavior.

Secondly, on the base of Eqs. (1) and (2) with U and D_2 from Eqs. (7) and (9) the time series of the index $S^*(t)$ has been generated. Its time evolution is presented Fig. 6 by the solid line. It has been obtained by averaging over 150 the simulated trajectories yielding the initial index decline greater than 25%. As it has been noted in the early observations [8] the aftercrash time behavior of the S&P500 index is characterized by an exponentially decaying sinusoidal function. In our case the generated data fit well enough with the function

$$S(t) = (A_1 e^{-\alpha_1 t} + A_2 e^{-\alpha_2 t}) \sin(\omega t + \gamma) + A_0, \quad (10)$$

incorporating the superposition of two the exponential functions with the different exponent α_1 and α_2 . (the dashed line in Fig. 6). The parameters of Eq. (10) have been defined by the least-squares method and given in Table III.

Thirdly, as follows from Eq.(9) at the moment of the crash volatility $\sim \sqrt{D_2}$ makes a shock and further decays to the aftercrisis level as power law with the exponent approximately equal to 0.3. The similar behavior has been also detected for the implied volatility of the S&P500 index (with exponent approximately equal to 1.) [8].

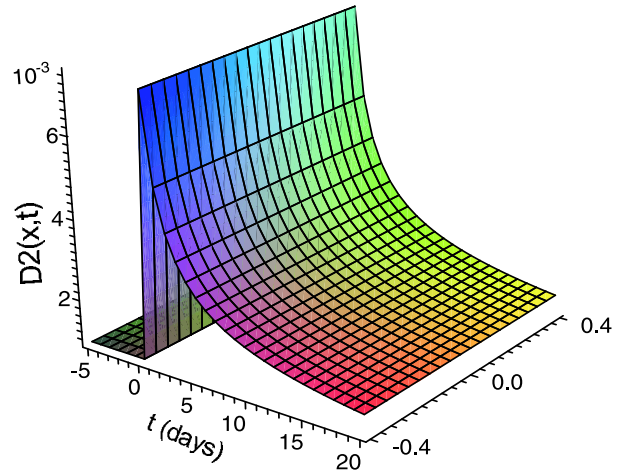


FIG. 5: The behavior of the diffusion coefficient D_2 given by Eq. (9).

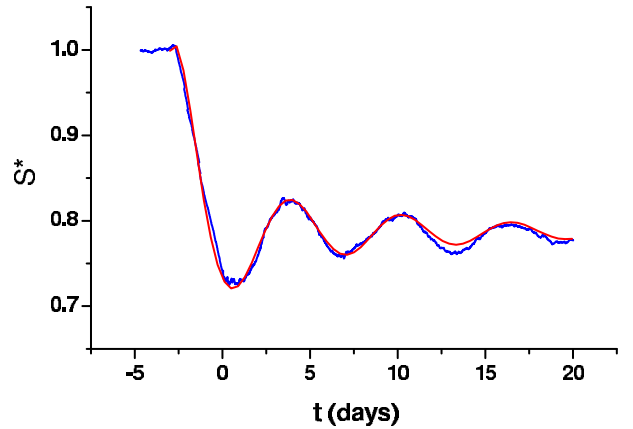


FIG. 6: The time evolution of the fictitious index $S^*(t)$. The solid line has been obtained by averaging over 150 the simulated trajectories yielding the initial index decline greater than 25%. The fit with Eq.(10) is shown by the dashed line.

Finally, we have examined the fulfillment of the Omori law within the framework of the given model. To this end the cumulative number $N(t)$ of the log-returns in interval $[0, t]$, whose absolute values exceed a given threshold value ℓ has been calculated for different t . As it is seen from Fig. 7 the generated data are described well enough by power law $t^{1-\Omega}$, that is characteristic for the Omori law, with $\Omega = 0.626$ for the threshold $\ell = \sigma$ and $\Omega = 0.704$ for $\ell = 1.5\sigma$ where $\sigma = 0.0685$ is the standard deviation of log-return $x(t)$ computed over the entire period of 25 days. The values of Ω are well consistent with the data found by Weber et al [14].

TABLE III: Parameters of Eq. (10)

A_0	A_1	A_2	α_1	α_2	ω	γ
0.787	0,05	0.031	0,09	0,64	1	- 2.41

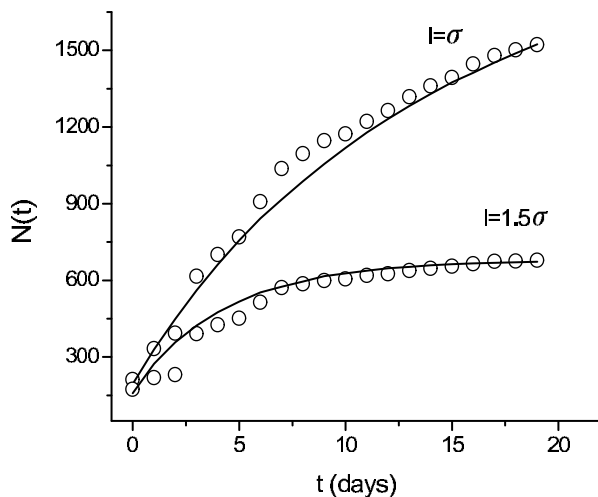


FIG. 7: Cumulative number $N(t)$ of log-returns in interval $[0, t]$, whose absolute values exceed a given threshold value ℓ for $\ell = \sigma$ and 1.5σ , where $\sigma = 0.0685$ is standard deviation of log-return $x(t)$ computed over the entire period of 25 days. The generated data are shown by circles, the solid lines are power law fits.

Thus the presented model, it seems, allows to describe the best known empirical facts relative to the aftercrash dynamics. To some extent this circumstance can serve as validation of performed estimation of the Kramers-Moyal coefficients.

In conclusion, the given work originates from the empirical observation that the large financial crashes are outliers and during the crash the statistic properties of a market are drastically changed in comparison with its typical properties. Based on these observations the physical model presents an overdamped Brownian particle with the variable noise intensity defined by the diffusion coefficient D_2 , moving in the nonstationary potential U . The market mechanism that triggers such potential is related to an imitation between traders increased up to a critical the point and a panic reigning in a market in the days of the crash. On the base of the empirical data within the scope of the Markowian approximation the Kramers-Moyal coefficients have been estimated with the subsequent determination of the potential $U(x, t)$. As it has been shown the given model reproduces well enough the best known empirical observations detected in the days of the large financial crash of October 1987.

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